Introduction

Think about all the angles formed by parallel lines intersected by a transversal. What are the relationships among those angles? In this lesson, we will prove those angle relationships. First, look at a diagram of a pair of parallel lines and notice the interior angles versus the exterior angles. The interior angles lie between the parallel lines and the exterior angles lie outside the pair of parallel lines. In the following diagram, line $k$ is the transversal. A transversal is a line that intersects a system of two or more lines. Lines $l$ and $m$ are parallel. The exterior angles are $\angle 1$, $\angle 2$, $\angle 7$, and $\angle 8$. The interior angles are $\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$. 

Prerequisite Skills

This lesson requires the use of the following skills:

- setting up and solving linear equations with a variable on both sides
- applying the Supplement Theorem
- applying the Vertical Angles Theorem
Key Concepts

- A straight line has a constant slope and parallel lines have the same slope.
- If a line crosses a set of parallel lines, then the angles in the same relative position have the same measures.
- Angles in the same relative position with respect to the transversal and the intersecting lines are corresponding angles.
- If the lines that the transversal intersects are parallel, then corresponding angles are congruent.

Postulate

**Corresponding Angles Postulate**

If two parallel lines are cut by a transversal, then corresponding angles are congruent.

Corresponding angles:

\[ \angle 1 \cong \angle 5, \quad \angle 2 \cong \angle 6, \quad \angle 3 \cong \angle 7, \quad \angle 4 \cong \angle 8 \]

The converse is also true. If corresponding angles of lines that are intersected by a transversal are congruent, then the lines are parallel.
**Alternate interior angles** are angles that are on opposite sides of the transversal and lie on the interior of the two lines that the transversal intersects.

If the two lines that the transversal intersects are parallel, then alternate interior angles are congruent.

**Theorem**

**Alternate Interior Angles Theorem**

If two parallel lines are intersected by a transversal, then alternate interior angles are congruent.

Alternate interior angles:

\[ \angle 3 \cong \angle 6, \quad \angle 4 \cong \angle 5 \]

The converse is also true. If alternate interior angles of lines that are intersected by a transversal are congruent, then the lines are parallel.
• **Same-side interior angles** are angles that lie on the same side of the transversal and are in between the lines that the transversal intersects.

• If the lines that the transversal intersects are parallel, then same-side interior angles are supplementary.

• Same-side interior angles are sometimes called consecutive interior angles.

**Theorem**

**Same-Side Interior Angles Theorem**

If two parallel lines are intersected by a transversal, then same-side interior angles are supplementary.

Same-side interior angles:

\[ m\angle 3 + m\angle 5 = 180 \]
\[ m\angle 4 + m\angle 6 = 180 \]

The converse is also true. If same-side interior angles of lines that are intersected by a transversal are supplementary, then the lines are parallel.
• **Alternate exterior angles** are angles that are on opposite sides of the transversal and lie on the exterior (outside) of the two lines that the transversal intersects.

• If the two lines that the transversal intersects are parallel, then alternate exterior angles are congruent.

**Theorem**

**Alternate Exterior Angles Theorem**

If parallel lines are intersected by a transversal, then alternate exterior angles are congruent.

Alternate exterior angles:

\[ \angle 1 \cong \angle 8, \quad \angle 2 \cong \angle 7 \]

The converse is also true. If alternate exterior angles of lines that are intersected by a transversal are congruent, then the lines are parallel.
• **Same-side exterior angles** are angles that lie on the same side of the transversal and are outside the lines that the transversal intersects.

• If the lines that the transversal intersects are parallel, then same-side exterior angles are supplementary.

• Same-side exterior angles are sometimes called consecutive exterior angles.

### Theorem

**Same-Side Exterior Angles Theorem**

If two parallel lines are intersected by a transversal, then same-side exterior angles are supplementary.

Same-side exterior angles:

\[
m \angle 1 + m \angle 7 = 180 \\
m \angle 2 + m \angle 8 = 180
\]

The converse is also true. If same-side exterior angles of lines that are intersected by a transversal are supplementary, then the lines are parallel.
- When the lines that the transversal intersects are parallel and perpendicular to the transversal, then all the interior and exterior angles are congruent right angles.

**Theorem**

**Perpendicular Transversal Theorem**

If a line is perpendicular to one line that is parallel to another, then the line is perpendicular to the second parallel line.

The converse is also true. If a line intersects two lines and is perpendicular to both lines, then the two lines are parallel.
Common Errors/Misconceptions

- setting expressions equal to each other instead of setting up expressions as a supplemental relationship and vice versa
- not being able to recognize the relative positions of the angles in a set of parallel lines intersected by a transversal
- misidentifying or not being able to identify the theorem or postulate to apply
- leaving out definitions or other steps in proofs
- assuming information not given in a diagram or problem statement that cannot be assumed
- assuming drawings are to scale
Example 1

Given $\overline{AB} \parallel \overline{DE}$, prove that $\triangle ABC \sim \triangle DEC$.

1. State the given information.
   $\overline{AB} \parallel \overline{DE}$

2. Extend the lines in the figure to show the transversals.
   Indicate the corresponding angles and mark the congruence of the corresponding angles with arcs.

   $\angle CAB \cong \angle CDE$ and $\angle CBA \cong \angle CED$ because each pair is a set of corresponding angles.
3. Use the AA (angle-angle) criteria.

When two pairs of corresponding angles of a triangle are congruent, the angles are similar.

In this case, we actually know that all three pairs of corresponding angles are congruent because $\angle C \cong \angle C$ by the Reflexive Property.

4. Write the information in a two-column proof.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $AB \parallel DE$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle CAB \cong \angle CDE$, $\angle CBA \cong \angle CED$</td>
<td>2. Corresponding Angles Postulate</td>
</tr>
<tr>
<td>3. $\triangle ABC \sim \triangle DEC$</td>
<td>3. AA Postulate</td>
</tr>
</tbody>
</table>
Example 2

Given two parallel lines and a transversal, prove that alternate interior angles are congruent. In the following diagram, lines $l$ and $m$ are parallel. Line $k$ is the transversal.

Given: $l \parallel m$, and line $k$ is a transversal.
Prove: $\angle 3 \cong \angle 6$

1. State the given information.
   $l \parallel m$, and line $k$ is a transversal.
2. Use the Corresponding Angles Postulate.

Corresponding angles are angles that lie in the same relative position with respect to the transversal and the lines the transversal intersects. If the lines that the transversal intersects are parallel, then corresponding angles are congruent. \( \angle 3 \) and \( \angle 7 \) are corresponding angles because they are both below the parallel lines and on the left side of the transversal.

\( \angle 3 \cong \angle 7 \) because they are corresponding angles.

3. Use the Vertical Angles Theorem.

Vertical angles are formed when a pair of lines intersect. Vertical angles are the nonadjacent angles formed by these intersecting lines. Vertical angles are congruent.

\( \angle 7 \cong \angle 6 \) because they are vertical angles.

4. Use the Transitive Property.

Since \( \angle 3 \cong \angle 7 \) and \( \angle 7 \cong \angle 6 \), \( \angle 3 \cong \angle 6 \).

5. Write the information in a flow proof.

- Given: \( l \parallel m \)
- Line \( l \) is a transversal.
- \( \angle 3 \cong \angle 7 \) (Corresponding Angles Postulate)
- \( \angle 7 \cong \angle 6 \) (Vertical Angles Theorem)
- \( \angle 3 \cong \angle 6 \) (Transitive Property)
Example 3

In the following diagram, $AB \parallel CD$ and $AC \parallel BD$. If $m \angle 1 = 3(x + 15)$, $m \angle 2 = 2x + 55$, and $m \angle 3 = 4y + 9$, find the measures of the unknown angles and the values of $x$ and $y$.

1. Find the relationship between two angles that have the same variable. $\angle 1$ and $\angle 2$ are same-side interior angles and are both expressed in terms of $x$.

2. Use the Same-Side Interior Angles Theorem. Same-side interior angles are supplementary. Therefore, $m \angle 1 + m \angle 2 = 180$. 
3. Use substitution and solve for $x$.

$$m\angle 1 = 3(x + 15) \text{ and } m\angle 2 = 2x + 55$$

$$m\angle 1 + m\angle 2 = 180$$

$$[3(x + 15)] + (2x + 55) = 180$$

$$3x + 45 + 2x + 55 = 180$$

$$5x + 100 = 180$$

$$5x = 80$$

$$x = 16$$

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Same-Side Interior Angles Theorem

Substitute $3(x + 15)$ for $m\angle 1$ and $2x + 55$ for $m\angle 2$.

Distribute.

Combine like terms.

Subtract 100 from both sides of the equation.

Divide both sides by 5.

4. Find $m\angle 1$ and $m\angle 2$ using substitution.

$$m\angle 1 = 3(x + 15) ; x = 16$$

$$m\angle 1 = 3(16 + 15)$$

$$m\angle 1 = 3(31)$$

$$m\angle 1 = 93$$

$$m\angle 2 = 2x + 55 ; x = 16$$

$$m\angle 2 = 2(16) + 55$$

$$m\angle 2 = 32 + 55$$

$$m\angle 2 = 87$$

After finding $m\angle 1$, to find $m\angle 2$ you could alternately use the Same-Side Interior Angles Theorem, which says that same-side interior angles are supplementary.

$$m\angle 1 + m\angle 2 = 180$$

$$(93) + m\angle 2 = 180$$

$$m\angle 2 = 180 - 93$$

$$m\angle 2 = 87$$

5. Find the relationship between one of the known angles and the last unknown angle, $\angle 3$.

$\angle 1$ and $\angle 3$ lie on the opposite side of the transversal on the interior of the parallel lines. This means they are alternate interior angles.
6. Use the Alternate Interior Angles Theorem.

The Alternate Interior Angles Theorem states that alternate interior angles are congruent if the transversal intersects a set of parallel lines. Therefore, \( \angle 1 \cong \angle 3 \).

7. Use the definition of congruence and substitution to find \( m \angle 3 \).

\[ \angle 1 \cong \angle 3, \text{ so } m\angle 1 = m\angle 3. \]

\[ m\angle 1 = 93 \]

Using substitution, \( 93 = m\angle 3 \).

8. Use substitution to solve for \( y \).

\[ m\angle 3 = 4y + 9 \quad \text{Given} \]
\[ 93 = 4y + 9 \quad \text{Substitute 93 for } m\angle 3. \]
\[ 84 = 4y \quad \text{Subtract 9 from both sides of the equation.} \]
\[ y = 21 \quad \text{Simplify.} \]
Example 4

In the following diagram, \( \overrightarrow{AB} || \overrightarrow{CD} \). If \( m\angle 1 = 35 \) and \( m\angle 2 = 65 \), find \( m\angle EQF \).

1. Draw a third parallel line that passes through point \( Q \).
   Label a second point on the line as \( P \). \( \overrightarrow{PQ} || \overrightarrow{AB} || \overrightarrow{CD} \).
2. Use $\overline{QE}$ as a transversal to $\overline{AB}$ and $\overline{PQ}$ and identify angle relationships.

$\angle 1 \cong \angle BEQ$ because they are vertical angles.

$\angle BEQ \cong \angle EQP$ because they are alternate interior angles.

$\angle 1 \cong \angle EQP$ by the Transitive Property.

It was given that $m\angle 1 = 35$.

By substitution, $m\angle EQP = 35$.

3. Use $\overline{QF}$ as a transversal to $\overline{PQ}$ and $\overline{CD}$ and identify angle relationships.

$\angle 2 \cong \angle FQP$ because they are alternate interior angles.

It was given that $m\angle 2 = 65$.

By substitution, $m\angle FQP = 65$.

4. Use angle addition.

Notice that the angle measure we are looking for is made up of two smaller angle measures that we just found.

$m\angle EFQ = m\angle EQP + m\angle FQP$

$m\angle EFQ = 35 + 65$

$m\angle EFQ = 100$